Appendix E

Interactions of Electrons and Photons with Matter

27. PASSAGE OF PARTICLES THROUGH MATTER

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27.1. Notation

Table 27.1: Summary of variables used in this section. The kinematic variables β and γ have their usual meanings.

Symbol	Definition	Units or Value
α	Fine structure constant ($e^2/4\pi\epsilon_0\hbar c$)	1/137.035 999 11(46)
M	Incident particle mass	MeV/c^2
E	Incident particle energy γMc^2	MeV
T	Kinetic energy	MeV
m_ec^2	Electron mass $\times$ c^2	0.510 998 918(44) MeV
r_e	Classical electron radius $e^2/4\pi\epsilon_0 m_ec^2$	2.817 940 325(28) fm
N_A	Avogadro's number	6.022 1415(10) $\times$ 10^{23} mol^{-1}
ze	Charge of incident particle	
Z	Atomic number of absorber	
A	Atomic mass of absorber	g mol^{-1}
K/A	$4\pi N_A r_e^2 m_e c^2/A$	0.307 075 MeV g^{-1} cm^2 for A = 1 g mol^{-1}
I	Mean excitation energy	eV (*Nota bene!*)
$\delta(\beta\gamma)$	Density effect correction to ionization energy loss	
$\hbar\omega_p$	Plasma energy ($\sqrt{4\pi N_e r_e^3}\ m_ec^2/\alpha$)	$28.816\sqrt{\rho\langle Z/A\rangle}$ eV$^{(a)}$
N_c	Electron density	(units of r_e)$^{-3}$
w_j	Weight fraction of the jth element in a compound or mixture	
n_j	$\propto$ number of jth kind of atoms in a compound or mixture	
—	$4\alpha r_e^2 N_A/A$	(716.408 g cm^{-2})$^{-1}$ for $A = 1$ g mol^{-1}
X_0	Radiation length	g cm^{-2}
E_c	Critical energy for electrons	MeV
$E_{\mu c}$	Critical energy for muons	GeV
E_s	Scale energy $\sqrt{4\pi/\alpha}\ m_ec^2$	21.2052 MeV
R_M	Molière radius	g cm^{-2}

$^{(a)}$ For ρ in g cm^{-3}.

$$= z_1\, x\, \theta_0/\sqrt{12} + z_2\, x\, \theta_0/2 \; ; \tag{27.18}$$

$$\theta_{\rm plane} = z_2\, \theta_0 \; . \tag{27.19}$$

Note that the second term for $y_{\rm plane}$ equals $x\,\theta_{\rm plane}/2$ and represents the displacement that would have occurred had the deflection $\theta_{\rm plane}$ all occurred at the single point $x/2$.

For heavy ions the multiple Coulomb scattering has been measured and compared with various theoretical distributions [34].

27.4. Photon and electron interactions in matter

27.4.1. ***Radiation length*** : High-energy electrons predominantly lose energy in matter by bremsstrahlung, and high-energy photons by e^+e^- pair production. The characteristic amount of matter traversed for these related interactions is called the radiation length X_0, usually measured in g cm^{-2}. It is both (a) the mean distance over which a high-energy electron loses all but $1/e$ of its energy by bremsstrahlung, and (b) $\frac{7}{9}$ of the mean free path for pair production by a high-energy photon [35]. It is also the appropriate scale length for describing high-energy electromagnetic cascades. X_0 has been calculated and tabulated by Y.S. Tsai [36]:

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A} \left\{ Z^2 \left[L_{\rm rad} - f(Z) \right] + Z\, L'_{\rm rad} \right\} . \tag{27.20}$$

For $A = 1$ g mol^{-1}, $4\alpha r_e^2 N_A/A = (716.408 \text{ g cm}^{-2})^{-1}$. $L_{\rm rad}$ and $L'_{\rm rad}$ are given in Table 27.2. The function $f(Z)$ is an infinite sum, but for elements up to uranium can be represented to 4-place accuracy by

$$f(Z) = a^2 \left[(1+a^2)^{-1} + 0.20206 \right.$$
$$\left. -0.0369\, a^2 + 0.0083\, a^4 - 0.002\, a^6 \right] , \tag{27.21}$$

where $a = \alpha Z$ [37].

Table 27.2: Tsai's $L_{\rm rad}$ and $L'_{\rm rad}$, for use in calculating the radiation length in an element using Eq. (27.20).

Element	Z	$L_{\rm rad}$	$L'_{\rm rad}$
H	1	5.31	6.144
He	2	4.79	5.621
Li	3	4.74	5.805
Be	4	4.71	5.924
Others	> 4	$\ln(184.15\, Z^{-1/3})$	$\ln(1194\, Z^{-2/3})$

Although it is easy to use Eq. (27.20) to calculate X_0, the functional dependence on Z is somewhat hidden. Dahl provides a compact fit to the data [38]:

$$X_0 = \frac{716.4 \text{ g cm}^{-2}\, A}{Z(Z+1)\ln(287/\sqrt{Z})} \; . \tag{27.22}$$

Results using this formula agree with Tsai's values to better than 2.5% for all elements except helium, where the result is about 5% low.

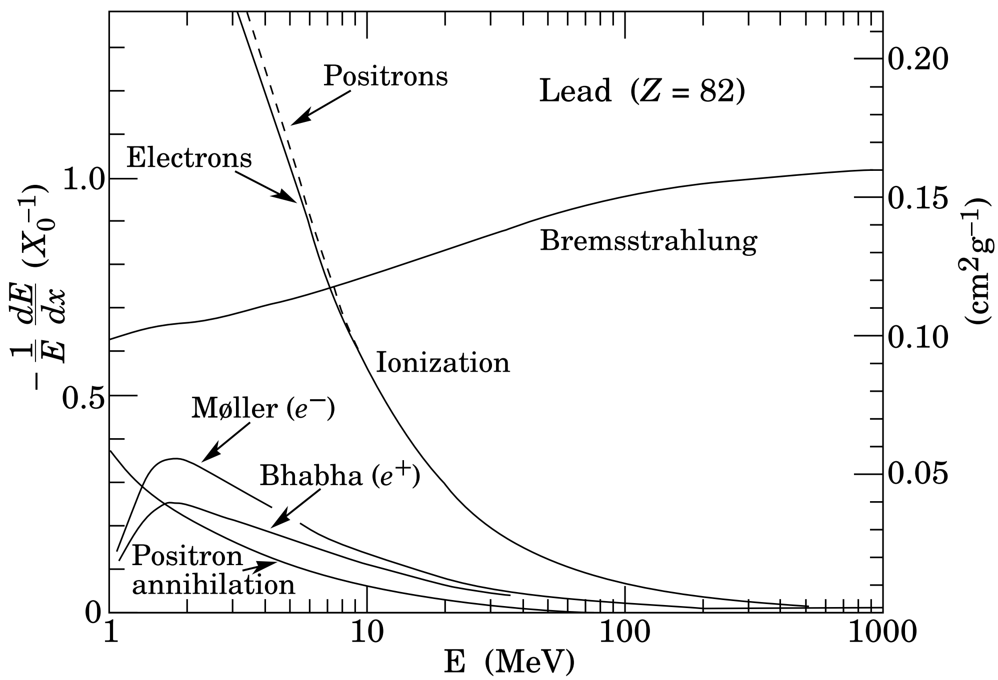

Figure 27.10: Fractional energy loss per radiation length in lead as a function of electron or positron energy. Electron (positron) scattering is considered as ionization when the energy loss per collision is below 0.255 MeV, and as Møller (Bhabha) scattering when it is above. Adapted from Fig. 3.2 from Messel and Crawford, *Electron-Photon Shower Distribution Function Tables for Lead, Copper, and Air Absorbers*, Pergamon Press, 1970. Messel and Crawford use $X_0(\text{Pb}) = 5.82\ \text{g/cm}^2$, but we have modified the figures to reflect the value given in the Table of Atomic and Nuclear Properties of Materials ($X_0(\text{Pb}) = 6.37\ \text{g/cm}^2$).

The radiation length in a mixture or compound may be approximated by

$$1/X_0 = \sum w_j/X_j \ , \tag{27.23}$$

where w_j and X_j are the fraction by weight and the radiation length for the jth element.

27.4.2. ***Energy loss by electrons*** : At low energies electrons and positrons primarily lose energy by ionization, although other processes (Møller scattering, Bhabha scattering, e^+ annihilation) contribute, as shown in Fig. 27.10. While ionization loss rates rise logarithmically with energy, bremsstrahlung losses rise nearly linearly (fractional loss is nearly independent of energy), and dominates above a few tens of MeV in most materials

Ionization loss by electrons and positrons differs from loss by heavy particles because of the kinematics, spin, and the identity of the incident electron with the electrons which it ionizes. Complete discussions and tables can be found in Refs. 7, 8, and 27.

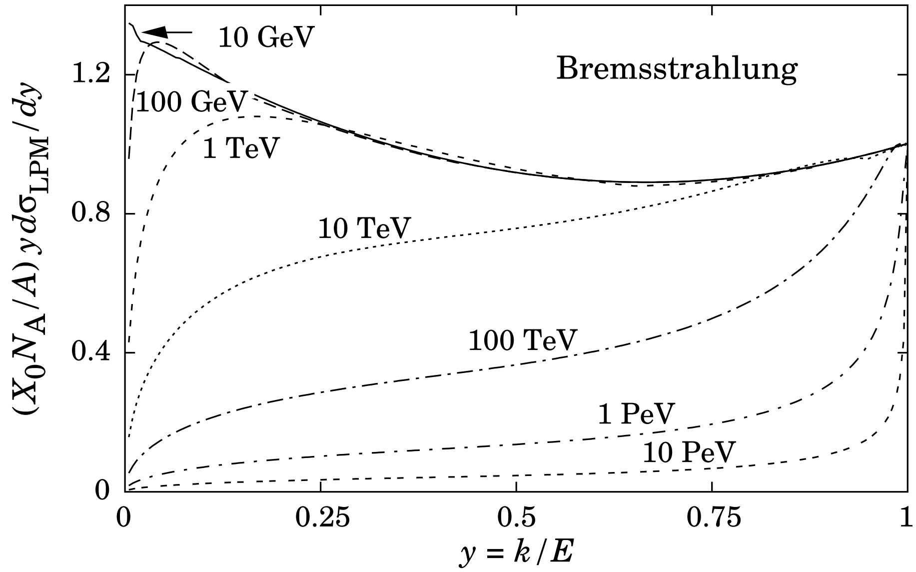

Figure 27.11: The normalized bremsstrahlung cross section $k\, d\sigma_{LPM}/dk$ in lead versus the fractional photon energy $y = k/E$. The vertical axis has units of photons per radiation length.

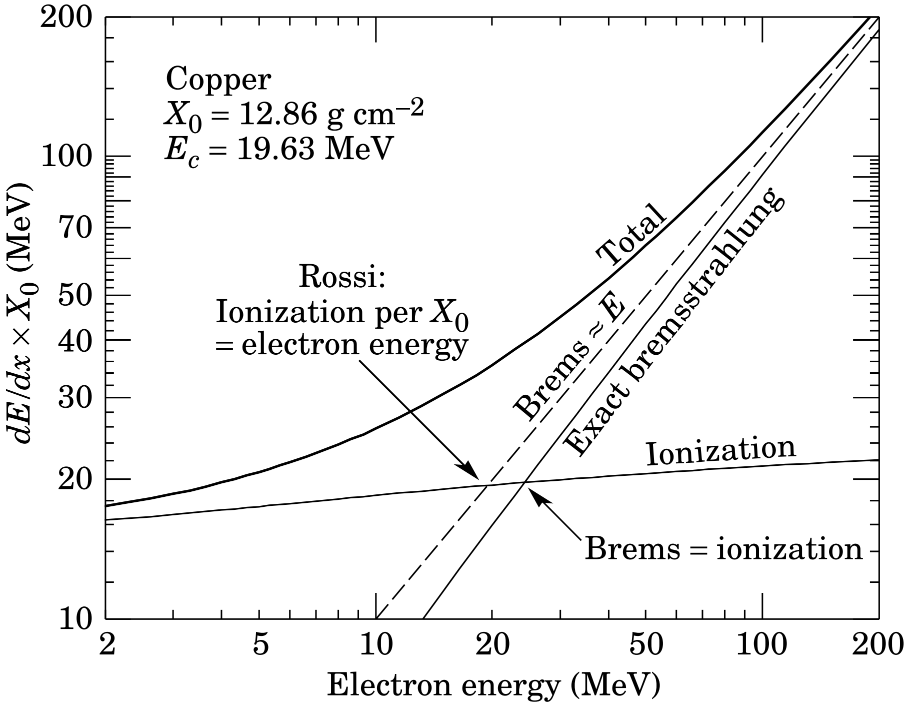

Figure 27.12: Two definitions of the critical energy E_c.

At very high energies and except at the high-energy tip of the bremsstrahlung spectrum, the cross section can be approximated in the "complete screening case" as [36]

$$d\sigma/dk = (1/k)4\alpha r_e^2\{(\tfrac{4}{3} - \tfrac{4}{3}y + y^2)[Z^2(L_{\rm rad} - f(Z)) + Z\,L'_{\rm rad}] \\ + \tfrac{1}{9}(1-y)(Z^2+Z)\}\ , \tag{27.24}$$

where $y = k/E$ is the fraction of the electron's energy transfered to the radiated photon. At small y (the "infrared limit") the term on the second line ranges from 1.7% (low Z) to 2.5% (high Z) of the total. If it is ignored and the first line simplified with the definition of X_0 given in Eq. (27.20), we have

$$\frac{d\sigma}{dk} = \frac{A}{X_0 N_A k}\left(\tfrac{4}{3} - \tfrac{4}{3}y + y^2\right)\ . \tag{27.25}$$

This cross section (times k) is shown by the top curve in Fig. 27.11.

This formula is accurate except in near $y = 1$, where screening may become incomplete, and near $y = 0$, where the infrared divergence is removed by the interference of bremsstrahlung amplitudes from nearby scattering centers (the LPM effect) [39,40] and dielectric suppression [41,42]. These and other suppression effects in bulk media are discussed in Sec. 27.4.5.

With decreasing energy ($E \lesssim 10$ GeV) the high-y cross section drops and the curves become rounded as $y \to 1$. Curves of this familar shape can be seen in Rossi [4] (Figs. 2.11.2,3); see also the review by Koch & Motz [43].

Except at these extremes, and still in the complete-screening approximation, the the number of photons with energies between $k_{\rm min}$ and $k_{\rm max}$ emitted by an electron travelling a distance $d \ll X_0$ is

$$N_\gamma = \frac{d}{X_0}\left[\frac{4}{3}\ln\left(\frac{k_{\rm max}}{k_{\rm min}}\right) - \frac{4(k_{\rm max}-k_{\rm min})}{3E} + \frac{(k_{\rm max}-k_{\rm min})^2}{2E^2}\right]\ . \tag{27.26}$$

27.4.3. ***Critical energy*** : An electron loses energy by bremsstrahlung at a rate nearly proportional to its energy, while the ionization loss rate varies only logarithmically with the electron energy. The *critical energy* E_c is sometimes defined as the energy at which the two loss rates are equal [44]. Berger and Seltzer [44] also give the approximation $E_c = (800\ {\rm MeV})/(Z + 1.2)$. This formula has been widely quoted, and has been given in older editions of this *Review* [45]. Among alternate definitions is that of Rossi [4], who defines the critical energy as the energy at which the ionization loss per radiation length is equal to the electron energy. Equivalently, it is the same as the first definition with the approximation $|dE/dx|_{\rm brems} \approx E/X_0$. This form has been found to describe transverse electromagnetic shower development more accurately (see below). These definitions are illustrated in the case of copper in Fig. 27.12.

The accuracy of approximate forms for E_c has been limited by the failure to distinguish between gases and solid or liquids, where there is a substantial difference in ionization at the relevant energy because of the density effect. We distinguish these two cases in Fig. 27.13. Fits were also made with functions of the form $a/(Z + b)^\alpha$, but α was found to be essentially unity. Since E_c also depends on A, I, and other factors, such forms are at best approximate.

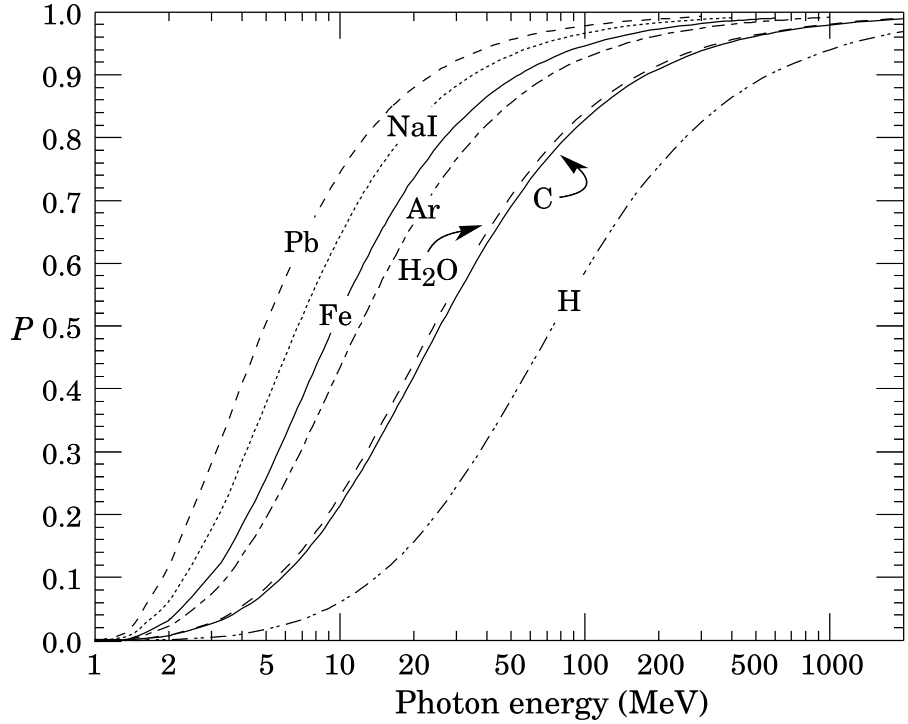

Figure 27.13: Electron critical energy for the chemical elements, using Rossi's definition [4]. The fits shown are for solids and liquids (solid line) and gases (dashed line). The rms deviation is 2.2% for the solids and 4.0% for the gases. (Computed with code supplied by A. Fassó.)

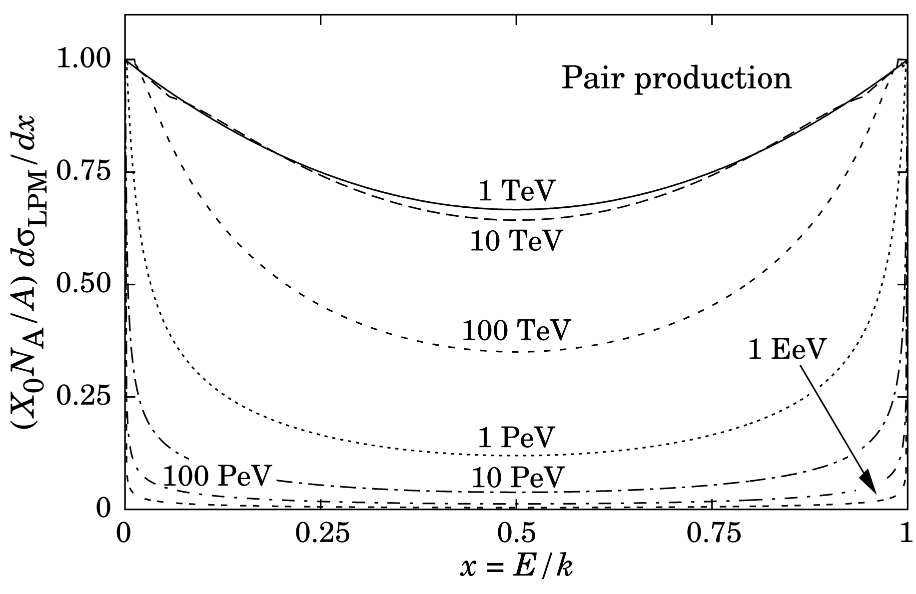

Figure 27.15: The normalized pair production cross section $d\sigma_{LPM}/dy$, versus fractional electron energy $x = E/k$.

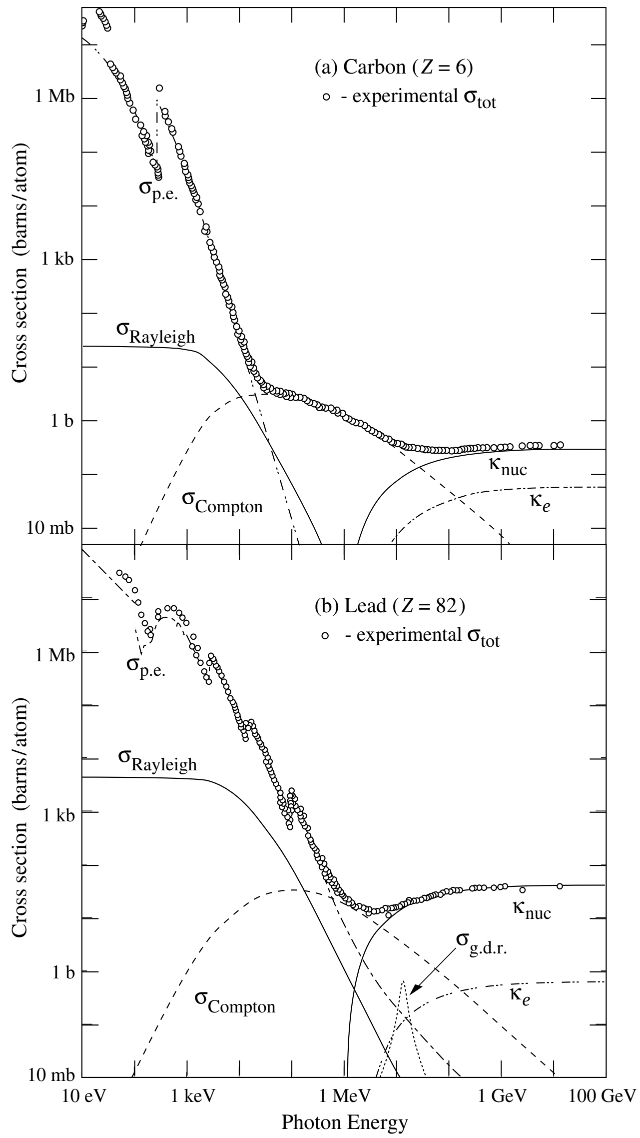

Figure 27.14: Photon total cross sections as a function of energy in carbon and lead, showing the contributions of different processes:

$\sigma_{\text{p.e.}}$ = Atomic photoelectric effect (electron ejection, photon absorption)

σ_{Rayleigh} = Rayleigh (coherent) scattering–atom neither ionized nor excited

σ_{Compton} = Incoherent scattering (Compton scattering off an electron)

κ_{nuc} = Pair production, nuclear field

κ_e = Pair production, electron field

$\sigma_{\text{g.d.r.}}$ = Photonuclear interactions, most notably the Giant Dipole Resonance [46]. In these interactions, the target nucleus is broken up.

Data from [47]; parameters for $\sigma_{\text{g.d.r.}}$ from [48]. Curves for these and other elements, compounds, and mixtures may be obtained from `http://physics.nist.gov/PhysRefData`. The photon total cross section is approximately flat for at least two decades beyond the energy range shown. Original figures courtesy J.H. Hubbell (NIST).

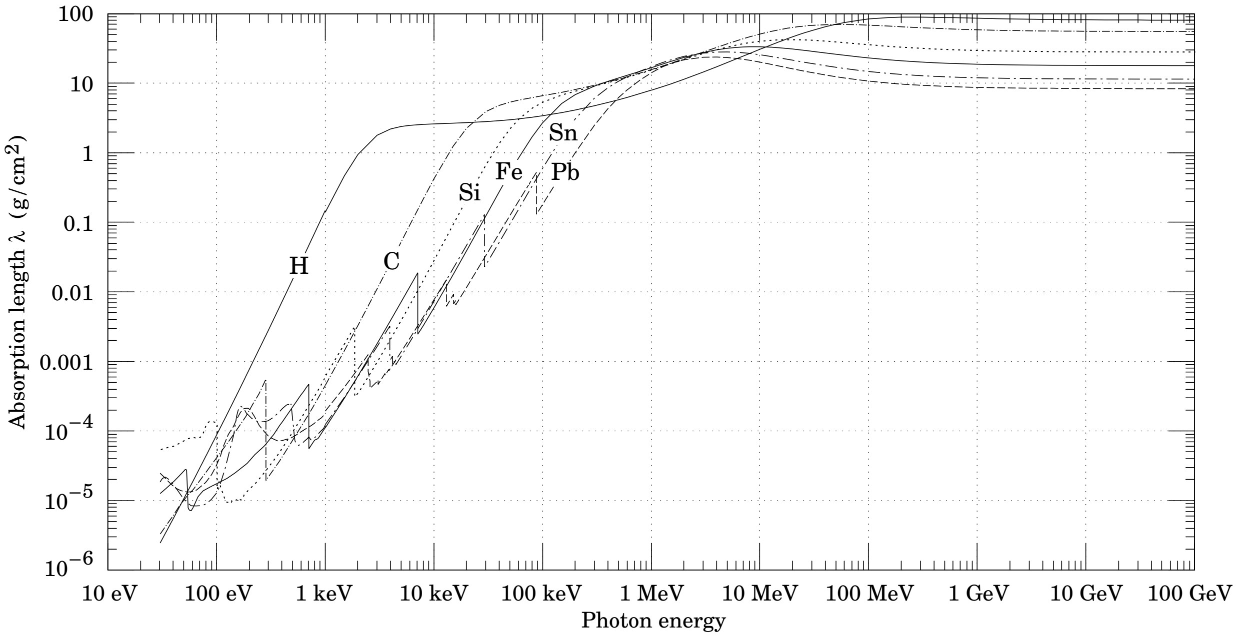

Figure 27.16: The photon mass attenuation length (or mean free path) $\lambda = 1/(\mu/\rho)$ for various elemental absorbers as a function of photon energy. The mass attenuation coefficient is μ/ρ, where ρ is the density. The intensity I remaining after traversal of thickness t (in mass/unit area) is given by $I = I_0 \exp(-t/\lambda)$. The accuracy is a few percent. For a chemical compound or mixture, $1/\lambda_{\rm eff} \approx \sum_{\rm elements} w_Z/\lambda_Z$, where w_Z is the proportion by weight of the element with atomic number Z. The processes responsible for attenuation are given in not Fig. 27.10. Since coherent processes are included, not all these processes result in energy deposition. The data for 30 eV $< E <$ 1 keV are obtained from `http://www-cxro.lbl.gov/optical_constants` (courtesy of Eric M. Gullikson, LBNL). The data for 1 keV $< E <$ 100 GeV are from `http://physics.nist.gov/PhysRefData`, through the courtesy of John H. Hubbell (NIST).

27.4.4. ***Energy loss by photons*** : Contributions to the photon cross section in a light element (carbon) and a heavy element (lead) are shown in Fig. 27.14. At low energies it is seen that the photoelectric effect dominates, although Compton scattering, Rayleigh scattering, and photonuclear absorption also contribute. The photoelectric cross section is characterized by discontinuities (absorption edges) as thresholds for photoionization of various atomic levels are reached. Photon attenuation lengths for a variety of elements are shown in Fig. 27.16, and data for 30 eV$< k <$100 GeV for all elements is available from the web pages given in the caption. Here k is the photon energy.

The increasing domination of pair production as the energy increases is shown in Fig. 27.17. Using approximations similar to those used to obtain Eq. (27.25), Tsai's formula for the differential cross section [36] reduces to

$$\frac{d\sigma}{dx} = \frac{A}{X_0 N_A}\left[1 - \tfrac{4}{3}x(1-x)\right] \tag{27.27}$$

in the complete-screening limit valid at high energies. Here $x = E/k$ is the fractional energy transfer to the pair-produced electron (or positron), and k is the incident photon energy. The cross section is very closely related to that for bremsstrahlung, since the Feynman diagrams are variants of one another. The cross section is of necessity symmetric between x and $1-x$, as can be seen by the solid curve in Fig. 27.15. See the review by Motz, Olsen, & Koch for a more detailed treatment [49].

Eq. (27.27) may be integrated to find the high-energy limit for the total e^+e^- pair-production cross section:

$$\sigma = \tfrac{7}{9}(A/X_0 N_A)\ . \tag{27.28}$$

Equation Eq. (27.28) is accurate to within a few percent down to energies as low as 1 GeV, particularly for high-Z materials.

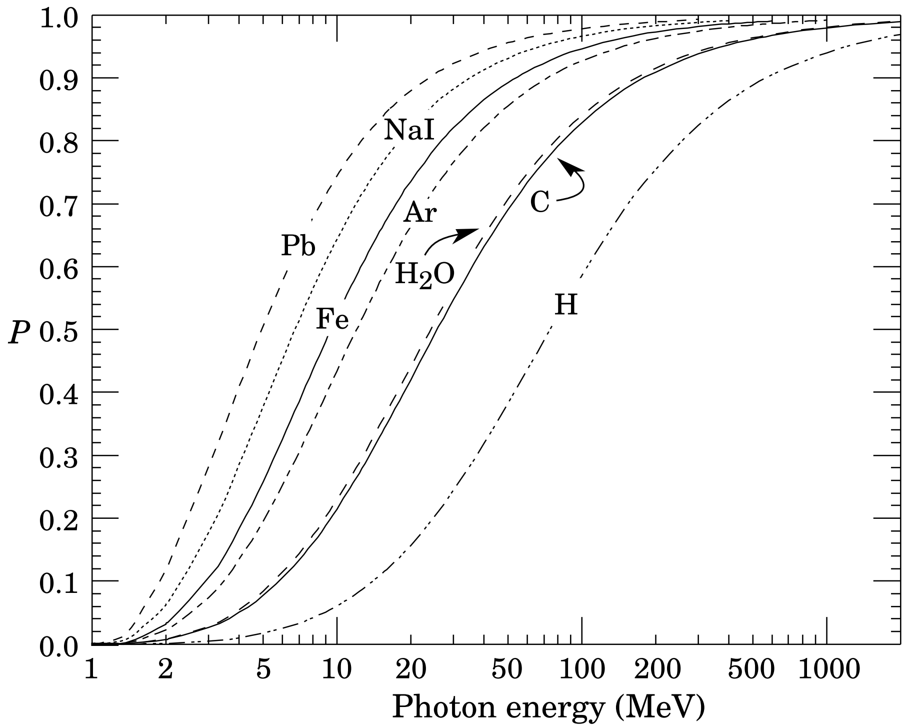

Figure 27.17: Probability P that a photon interaction will result in conversion to an e^+e^- pair. Except for a few-percent contribution from photonuclear absorption around 10 or 20 MeV, essentially all other interactions in this energy range result in Compton scattering off an atomic electron. For a photon attenuation length λ (Fig. 27.16), the probability that a given photon will produce an electron pair (without first Compton scattering) in thickness t of absorber is $P[1-\exp(-t/\lambda)]$.